Why must when computed in the induction equation

The strong conservative form of the induction equation is

Consider a magnetic field of . We may write our equation, with source terms as

Now consider the solution after the very first step. Note that, since there are no gradients in the induced magnetic field, we may write our equation as

It is very clear that if the velocity is not divergence free in the induction equation, that there will be significant non-zero divergence in the magnetic field.

For uniform properties, we have

Let's take the divergence of this